Part A – Opinion & Influence (3/4)

Selection

COMS 6998-1: Economic Dynamics in Social Networks
Wednesday, October 9th
Outline of Part A

- Intro: Life under the influence
- Consensus and Social Learning (Lec. 1)
- What’s against consensus?
  - Manipulation: self-interested nodes (Lec. 2)
  - Bias: homophily, selection, polarization (Lec. 3)
- Influence for adoption, spread
  - How to model? How to leverage? (Lec. 4)
In a nutshell

○ So far, agents consider information genuinely
  - Can be biased or stubborn (private information)
  - Treat other’s data with an exogenous trust
  - Can create persistent disagreement/fluctuation

○ But in reality information and trust co-evolve
  - What you think have influenced who you know
  - What you think may influence what you deduce
  - What you think influence who you know
WYT HAVE Influenced who you know

- Homophily
  - Very well known principle
  - How to model it?
- Typically, \( x_i(0) \) not chosen independently
- Multi-type random networks
  - Various classes \( k=1,2,\ldots \) of size \( n_1, n_2, \ldots \)
  - \( P[i\sim j] = P_{kl} \) for \( i \) of type \( k \) and \( j \) of type \( m \)
  - Introduce “incident” matrix (fraction of edges)

\[
F_{k,l}(P,n) = n_l P_{k,l} / \sum_m n_m P_{k,m}
\]
Homophily in Naïve Learning

Multi-type Random Network, let \( h = \lambda_2(F(P, n)) \)

Cvgce time \( \sim \log(n)/\log(1/h) \)

WYT Influence what you deduce

- How do people interpret
  - Raw facts
  - Additional information and context

- Lord, Ross, Lepper Experiment
  - Entry Survey on “Death penalty” proponent/opponents
  - Presented evidence on the deterrence effect
  - Phase 1: raw fact, phase 2: detailed information
  - Exit survey and measure of the difference
Examples

Kroner and Phillips (1977) compared murder rates for the year before and the year after adoption of capital punishment laws. Palmer and Crandall (1977) compared murder rates in 10 pairs of neighboring states with different capital punishment laws. In 8 of the 10 pairs, murder rates were higher in the state with capital punishment. This research opposes the deterrent effect of the death penalty.
Biased assimilation

1. Raw facts have non-linear effects
2. More information have polarizing effects

WYT influence who you know

- You actually take information genuinely into account but
  - Only if it fits in a bounded confidence interval
  - Alternatively, a state dependent connectivity

- Let us consider the simplest case
  - ONLY your state influences your connectivity
  - Equivalent to the nodal/pairwise updates:

\[
\begin{align*}
x_i & \leftarrow \frac{\sum_{j \in N_i} \mathbb{1}_{||x_j - x_i|| \leq 1} x_j}{\sum_{j \in N_i} \mathbb{1}_{||x_j - x_i|| \leq 1}} \\
\end{align*}
\]

\[
x_i, x_j \leftarrow \frac{x_i + x_j}{2} \quad \text{if } ||x_i - x_j|| \leq 1
\]
A typical evolution

THM: \( |x_i - x_j| > 1 \) implies they will never reconnect

Main Results

- THM: The nodal update dynamics always converge to a static fragmentation of opinion
  - Proof (1995 by Lorenz)
  - Many generalizations (Influence system)
  - It does so in a polynomial number of steps!

Lorenz’s result of convergence

- System evolves as $x(t + 1) = A(t, x(t)) \cdot x(t)$
  - Note that $A$ depends on $x(t)$
  - Or, $x(t) = A(t)x(t)$, where $A(t)$ by recurrence
    $$A(t + 1) = A(t, A(t)x(0))$$

- THM: Let $A(t)$ a seq. of row-stochastic mat.
  (i) self-confidence: $A_{i,i}(t) > 0$
  (ii) mutual-confidence: $A_{i,j}(t) > 0 \iff A_{j,i} > 0$
  (iii) minimal transfer: $A_{i,j}(t) > 0 \implies A_{i,j} > \delta$

$\lim A(t)$ is block diagonal, with identical rows

Influence system (long & technical)

- A system is bidirectional and diffusive if 
  \[ x_i(t + 1) \in [m_i + \rho(M_i - m_i); M_i - \rho(M_i - m_i)] \]
  where \( m_i = \min\{x_j \mid j \in N_t(i)\} \), \( M_i = \max\{x_j \mid j \in N_t(i)\} \)

- Where \( G(t) \) undirected but otherwise arbitrary

- We define \( E(s) = \sum_{t \geq 0} \sum_{(i,j) \in G_t} \|x_j - x_i\|_2^s \).

- THM: This total s-energy series is bounded!

  \[ \exists a > 0 \Rightarrow E(1) \leq \rho^{-an} \]
  \[ \forall 0 < s < 1 \Rightarrow \exists b > 0 \Rightarrow E(s) \leq s^{1-n} \rho^{-n^2-b} \]

A tighter bound for a specific case

- Assume evolution with bounded confidence
- THM: if opinion in $\mathbb{R}$, converges in $O(n^3)$ steps
  - If opinion in $\mathbb{R}^d$, converges in $O(n^{10}d^2)$ steps
- Proof use structure and local progress
  - STEP1: Study $V(t) = \sum_{i,j} (\|x_i - x_j\|^2 \wedge 1)$ show it only decreases at every step at least by $4 \sum_i \|x_i(t+1) - x_i(t)\|^2$
  - STEP2: If system not converged yet, either two agents merge, or one agent move by $c/(dn^4)$