Part A – Opinion & Influence (1/4)

Influence & Consensus

COMS 6998-1: Economic Dynamics in Social Networks
Wednesday, September 11th
Outline of Part A

- Intro: Life under the influence

- Consensus and Social Learning (Lec. 1)
  - What’s against consensus
    - Manipulation: forceful and self (Lec. 2)
    - Bias: homophily, selection, polarization (Lec. 3)

- Influence for adoption, spread
  - How to model? How to leverage? (Lec. 4)
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Some terminology
Epidemics

- Etymology: επί ‘upon’ + δῆμος ‘people’
  - (1) spread from people to people accidentally
  - (2) significant size (i.e. constant fraction)
- Stronger variants:
  - endemic=resides; pandemic=everyone reached
  - shaped history, shapes our present:
    Black plague (XIIIth),
    America (XVIth),
    AIDS (XXth)
Influence

- Etymology: *in* ‘into’ + *fluere* ‘to flow’
  - (1) affect development, behavior of someone
  - (2) typically opposed to individual free choice

- Much broader: diffusion, spread, cascades
  - Not necessarily dramatic: small effects matter
  - Not necessarily negative: innovation
  - Not necessarily accidental: more complex dynamics, conscious decision
Opinion

- **Etymology:** *opinio* (think, believe)
  - (1) It’s about knowledge, conscious decision
  - (2) It may or may not be related to truth/action

- **Surprising challenging to understand**
  - Bayesian: Inference only limited by information
  - Non Bayesian, or naïve, or rule-following: inference typically simplified
Their characteristic features?

- A structure (e.g. a graph) connecting agents
  - Directed/Undirected/Signed/Weighted,
  - Static/Dynamic/Opinion dependent
- Some states and some updating rules
  - Actions/Truth/Information (discrete/cont.)
  - Synchronous vs. asynchronous (discrete/cont.)
  - Homogeneous or heterogeneous nodes

- THMs: Structural prop. → Outcome, Prop
Different motivations

- **Economists:**
  - Which social process gather information for collective decision, and when does it succeed?

- **Sociologists:**
  - Are observations reproducible?
  - Are there mathematical laws to predict them?

- **Engineers, App developers:**
  - How to control systems? Should you?
  - How to use influence to one’s advantage?
Some examples of influence
Example 1: Asch’s experiment

Which segment would you say has length closest to left?

– What if N other people answered ‘A’ before you?

Group-pressure, conformity

Similar cascading effect

– Roman votes (centuries)

Effects of group pressure upon the modification and distortion of judgment, S. Asch, *Groups Leader. M.* (1951)
Example 2: Happiness

- Happy/unhappy clusters
- Influence of peers
- Seems to follow the nature of the ties

Similar results for spread of obesity, quit smoking

Dynamic spread of happiness in a large social network
J Fowler, N Christakis, BMJ (2008)
Example 3: Innovation

The diffusion of hybrid seed corn in two Iowa communities, B. Ryan, N. Gross, Rural sociology (1943)
Example 3: Innovation

Adoption of new medicine + physicians references or ties
- Doctors cited more often adopt at quicker rate

Index of pair-simultaneity
- Time between adoption of 2 in a pair vs. random
- Innovator, advisor, friend

The Diffusion of an Innovation Among Physicians
J Coleman, E Katz, H Menzel, Sociometry (1957)
Example 3: Innovation

Still a pressing need

– Developmental economics
  Use of fertilizer in developing regions
  “How can we induce people to make behavior changes that are known to have large and positive health benefits?”

Nudging Farmers to Use Fertilizer,
E. Duflo, M. Kremer, and J. Robinson, MIT (2010)
The Social Cure

- Can using Facebook to spread HIV Testing inside population
- Who can influence each other’s best
- Does it depend on age, gender, relationship status

https://www.youtube.com/watch?v=oV6HpiUgW2o
Effect of age, relationship status

Experiments on influence in FB Apps (movies)

Influence (dark), susceptible (light)

Younger are susceptible, married are not

Dyadic influence

Experiments on influence in FB Apps (movies)

Dyadic influence
Female influences male more than females

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A consensus pb

- Assume you need to make a difficult decision
  - Gather a bunch of experts in a meeting
  - Various backgrounds, experiences
  - Ask them to come prepared with a belief $x_i$

- For simplicity, let’s assume result in $[0;1]$
  - A priori, answer should be $\hat{s} = \sum_i s_i x_i$
  - How to choose weights to avoid protest?
Key idea

- A priori all you got is \((x_1, \ldots, x_n)\)!
- But the group collectively knows more
  - Experts value and trust some other
  - Assuming each expert \(i\) values \(j\)'s input \(\pi_{i,j}^0\)

- What you want:
  - Expert should reveal this value implicitly
  - Expert should not contest the result
Consensus Algorithm

− Let us normalize
− Let us iterate

\[ x^1_1 = \sum_j \frac{\pi^0_{i,j}}{\pi^1_{i,i}} x^0_j \quad \text{with} \quad \pi^1_i = \sum_j \pi^0_{i,j}. \]

Assumption: stated opinion does not change trust

Persuasion Bias

Everyone initially

\[ x_i^0 = \theta + \epsilon_i \]

Intrinsic trust \( \pi_i^0 = 1/E[\epsilon_i^2] \)

i listens to j is denoted \( q_{i,j} = 1 \)

After 1 round, it updates

\[
x_i^1 = \sum_j q_{ij} \pi_{i,j}^0 x_j^0 \quad \text{with} \quad \pi_i^1 = \sum_j q_{ij} \pi_{i,j}^0.
\]

Persuasion Bias

Or, equivalently

\[ x^1 = Tx^0 \]

where

\[ T_{i,j} = q_{i,j} \frac{\pi_{0,i,j}}{\pi_{1,i,i}} \]

or,

\[ T(\lambda_t) = \lambda_t \times I + (1 - \lambda_t) \times T \]

Assump: persuasion bias

Next information received treated as new signal (naïve)

Main results

○ THM1: Iterated consensus works!
  - Everyone’s stated opinion converge
  - The limit given by spectral property

○ THM2-3: Consensus \(\neq\) Rational estimator
  - Both are equal iff \(\sum_j q_{j,i} T_{j,j} = 1\)

○ Lesson: Consensus is not hard to reach,
  - but it’s also easy to bias through persuasion
Main results

- THM4: All dimensions collapse!
  - Assuming $x_i$ is in $\mathbb{R}^L$, we define
    $$d_{i,l}^t = \frac{x_{i,l}^t - \bar{x}_l^t}{\Delta_l^t}$$
    where $\Delta_l^t = \frac{1}{N} \sum_j x_{i,l}^t - \bar{x}_l^t$
    where $\bar{x}_l^t = \frac{1}{N} \sum_j x_{j,l}^t$

  - For large $t$, the discordance of all nodes lie in the same 1-dimensional space
    $$\lim_{t \to \infty} d_{i,l}^t = \phi_i \times D_l$$

- Lessons: complex issues becomes bipolar
Proof of THM1

- Simplification: assume $\lambda_t = \lambda > 0$

$$T = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

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A rather lukewarm conclusion

- A natural process builds consensus
  - But this consensus cannot be trusted!
  - And no matter how we argue, the debate ends up opposing two factions

- What’s next?
Wisdom of the crowd

West of England Fat stock and poultry exhibition. Guess the ox’s weight 787 participated (distribution above)

Actual weight: 1198lbs.

Can we reconcile these observations

- Maybe as society gets larger?
  - A sequence of large and larger networks
  - Creating a sequence of matrix \((T_n)_{n>0}\)

- Assuming nodes initial guess \(x_i^0(n)\)
  - Independent, with mean \(\theta\)
  - We wish, that or any error \(\varepsilon > 0\)

\[
\lim_{n \to \infty} P\left[ \max_i |x_i^{(t=\infty)}(n) - \theta| > \varepsilon \right] \to 0.
\]
Conditions on sequence

Let \( T_{B,C} = \sum_{i \in B, j \in C} T_{i,j} \)

\( T_n \) is balanced: \( \exists j_n \to \infty \)

\( \sup_n \frac{T_{B_n,C_n}^n}{T_{B_n,B_n}^n} < \infty \) for any \( B_n \) such that \( |B_n| \leq j_n \)

\( T_n \) is minimal dispersed: \( \exists q, r > 0 \)

\( T_{B_n,C_n}^n > r \) whenever \( |B_n| \geq q \) and \( |C_n|/n \to 1 \).

Main results

- THM: $T_n$ balanced and minimally dispersed implies that the wisdom of the crowd occurs
  - A sufficient and necessary conditions is
    $$\lim_{n \to \infty} \max_i s_i(n) \to 0$$
  - This does not occur when a finite and uniformly prominent family exists: $\sup_n |B_n| < \infty$
    $$\exists \alpha \text{ such that } \forall n, \exists t_n, \forall i \notin B, T_{t_n}(n)_{i,B} \geq \alpha$$
  - As an example:
In summary

- Consensus occurs as agents follow unbiased averaging rules
  - And outcome follows spectral property of graph
  - Speed of convergence as well (not shown)

- Outcome not “optimal/correct” but it can be
  - Main obstacle: disproportionate concentration of trust in a few nodes

- Is that too good to be true?
  - Unbiased agents with common goal/truth, continuous opinion, synchronization
Related: Gossip Algorithms

○ Why not use local iteration as an algorithmic principle
  - Gossip, Rumor mongering, Epidemic, Dist. Averaging
  - Applications to database, networking, dist. computing, control
  - Main difference: (1) Graph and interaction can be designed, typically asynchronous (2) speed of convergence matters.

○ Classic papers and survey
Related: Gossip Algorithms (cont’d)

- **Some recent papers:**
  - How to deal with nodes of large degree?
  - How to deal with evolving graphs?
Related: Bayesian learning

- Situation in which Bayes rule can be followed
  - Sequence of agents make irreversible actions
  - Actions are public knowledge
  - Classic result: herd behavior, “Discarding your private info becomes rational”

Related: Bayesian learning (cont’d)

○ What if observe only some past actions?
  – Under some diversity property, herding is avoided


○ What if people send signal to each other?